# ESTRUCTURAS PLANAS.

## T1 SISTEMAS PLANOS DE BARRAS ARTICULADAS (1h)

- GENERALIDADES
- HIPÓTESIS BÁSICAS PARA SU ANÁLISIS
- SISTEMAS ARTICULADOS ISOSTÁTICOS E HIPERESTÁTICOS
- DETERM. DEL GRADO DE HIPERESTATICIDAD
- ESTRUCTURAS ESTÁTICAMENTE DETERMINADAS (ISOSTÁTICAS)
- INDEFORMABILIDAD EXTERNA (GEOMÉTRICA)
- ISOSTATISMO INTERNO
- TIPOLOGÍA ESTRUCTURAL

## T2 CÁLCULO DE ESTRUCTURAS PLANAS ARTICULADAS ISOSTÁTICAS (9h)

- DETERMINACIÓN DE LAS REACCIONES
- CÁLCULO DE LOS ESFUERZOS EN LAS BARRAS
  - GRÁFICO.- CREMONA
  - ANALÍTICO.-
    - RITTER.-
    - MATRICIAL.-
- ESTRUCTURAS CON CARGAS FUERA DE LOS NUDOS
- EJERCICIOS

## T3 CÁLCULO DE ESTRUCTURAS PLANAS ARTICULADAS HIPERESTÁTICAS (4h)

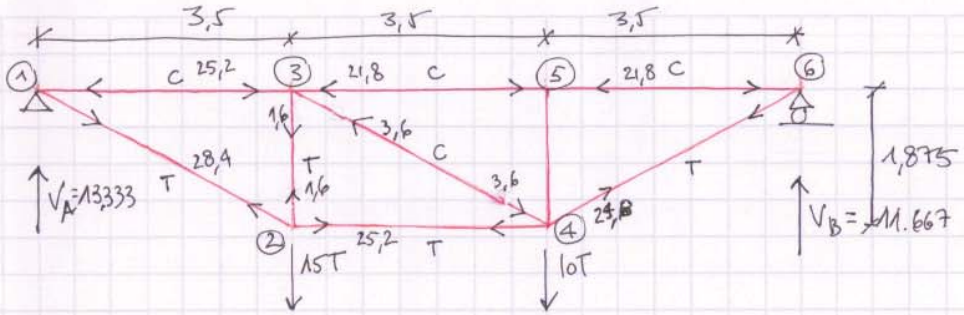
- ENERGÍA ELÁSTICA
- TEOREMA DE CASTIGLIANO
- CÁLCULO DE ESTRUCTURAS HIPERESTÁTICAS
  - HIPERESTÁTICAS INTERNAS
  - " " EXTERNAS.
- EJERCICIOS

## T4 CÁLCULO DE LOS DESPLAZAMIENTOS (2h)

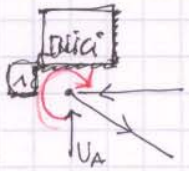
- PROCEDIMIENTO ANALÍTICO

## T5 ESTRUCTURAS ARTICULADAS Y EMPOTRADAS (4h)

- TIPOLOGÍA
- ACCIONES HORIZONTALES SOBRE LA CUBIERTA
- ACCIONES VERTICALES SOBRE LA CUBIERTA
- ACCIONES HORIZONTALES SOBRE LOS PILARES




$V_B = \frac{15 \cdot 3{,}5 + 10 \cdot 7}{10{,}5} = 11.667\,Kp.$ $V_A = 25 - V_B = 13{,}33\,Kp$




1-2 → $V_A$ → 3-1

2 | 2-1 → 2-3 → 2-4 → 15t.
28,4 1,6 25,2

3 | 3-2 → 3-1 → 3-5 → 3-4
1,6 25,2 21,8 3,6

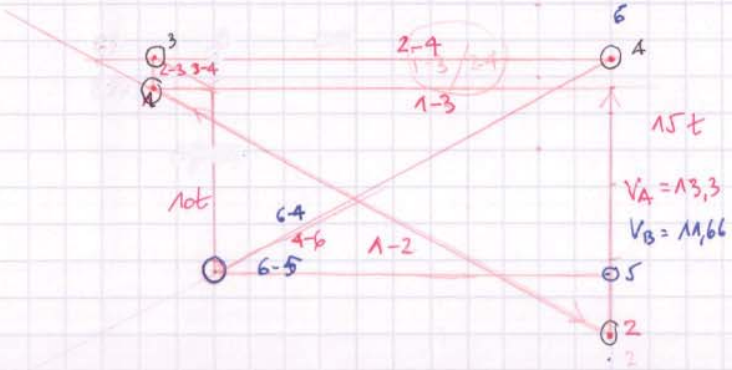
per simetria → en nus 5 ⇒ (5-4) = 0 !

Comprovacions ⇒ nusos (4) i (6)

4 | 4-2 → 4-3 → 4-5 → 4-6
25,2 3,6 24,8

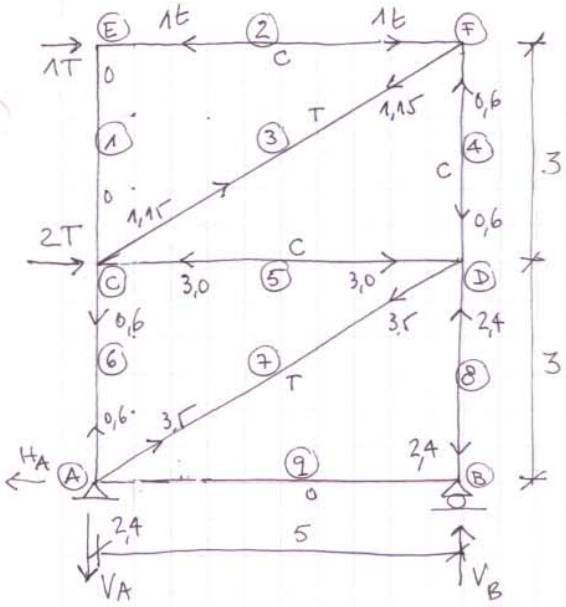
6 | 6-4 → 6-5 → $V_B$
24,8 21,8 11,66

1cm = 4000 kg
1cm = 4T



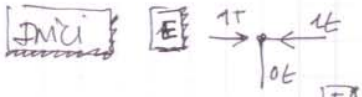

T2 CÀLCUL DE E.P.A.I. (CREMONA)

CREMONA (2.1)

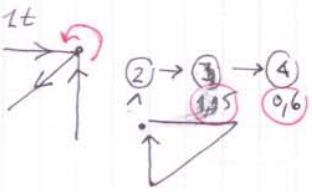



$$V_B = \frac{(1\cdot 6)+(2\cdot 3)}{5} = \frac{12}{5} = (2{,}4\,t) = V_A \qquad H_A = 3$$

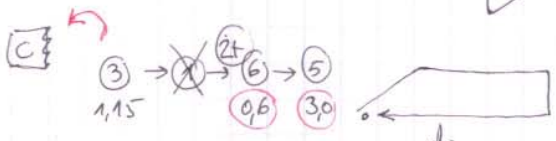
DMCI



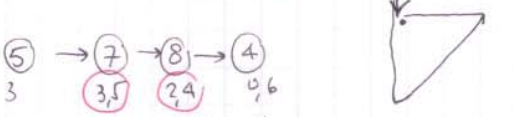

$1T_u = 2cm$




C

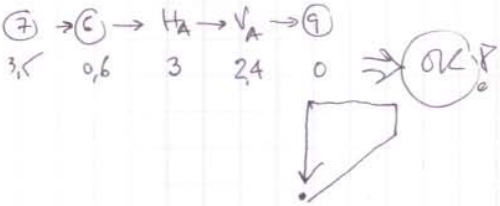


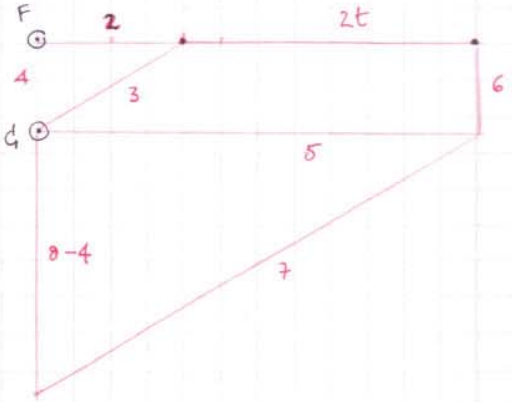

D



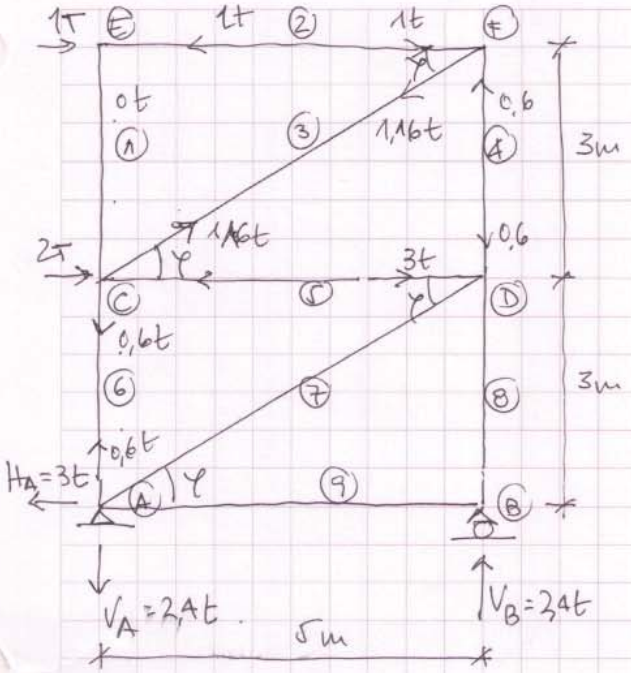

DIRETTE!

A



# ANALITIC (2.2)




$tg\varphi = \frac{3}{5} = 0{,}6 \Rightarrow \varphi = arctg\, 0{,}6 = \boxed{30{,}96°}$


$\sin\varphi = \frac{V}{D} ; = 0{,}514$

$\cos\varphi = \frac{H}{D} ; = 0{,}857$

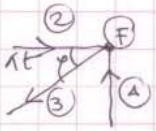
$tg\varphi = \frac{V}{H} ; = 0{,}6$

**NUDO E**


② = 1t (C)

① = 0t

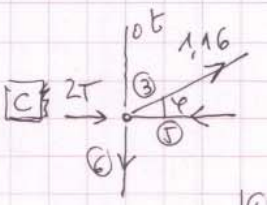
**NUDO F**




$③ = \frac{1t}{\cos\varphi} = 1{,}167\,t\ (T)$

④ = 1,167 · sen φ = 0,6 t (C)

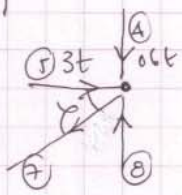
**NUDO C**




⑤ = 1,167 · cos φ + 2t = 3 t (C)

⑥ = 1,167 · sen φ = 0,6 t (T)

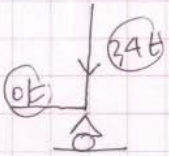
**NUDO D**

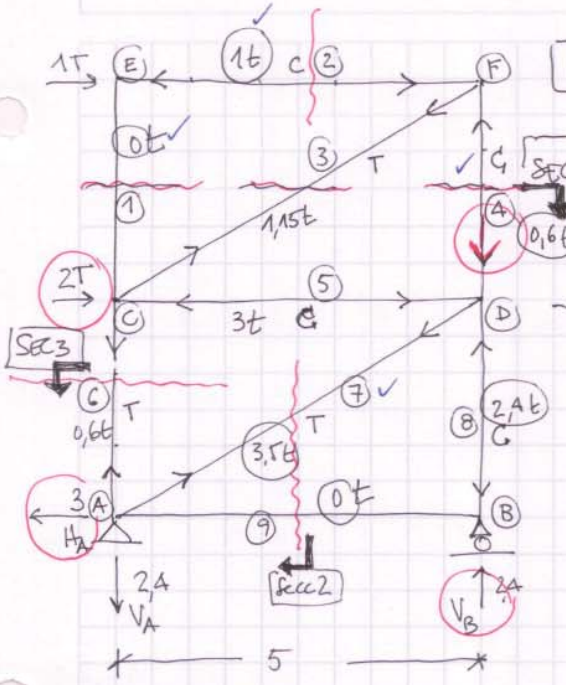



$⑦ = \frac{3t}{\cos\varphi} = 3{,}5\,t\ (T)$

⑧ = 3,5 · sen φ + 0,6 = 3,4 t (C)

**NUDO B**






M. RITTER

$\Sigma M_G = 0 \quad (B_4 \cdot 5) + (3 \cdot 3) = 2{,}4 \cdot 5$

$B_4 = \frac{(2{,}4 \cdot 5) - (3 \cdot 3)}{5} = \frac{3}{5} = 0{,}6\ t.$ (OK)

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SECC. 2. $\Sigma M_C = 0$

$(1 \cdot 3) + (3 \cdot 3) = (1 \cdot 3) + (B_7 \cdot 2{,}6)$

$B_7 = \frac{9}{2{,}6} = 3{,}47\ t \simeq 3{,}5\ t$ (OK)

$\Sigma M_D = 0 \quad (1 \cdot 3) + (3 \cdot 3) + (B_3 \cdot 2{,}6) = (1 \cdot 3) + (2{,}4 \cdot 5)$

$B_3 = \frac{(2{,}4 \cdot 5) - (3 \cdot 3)}{2{,}6} = \frac{3}{2{,}6} = 1{,}15\ t$ (OK)

$\Sigma M_A = 0$

$(1 \cdot 6) + (2 \cdot 3) + (1{,}15 \cdot 2{,}6) = (1 \cdot 6) + (B_5 \cdot 3) \quad ; \quad B_5 = \frac{(2 \cdot 3) + (1{,}15 \cdot 2{,}6)}{3} = 3\ t$ (OK)

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SECCIÓ 3 $\quad \Sigma M_D = 0 \quad (3 \cdot 3) + (B_6 \cdot 5) = 2{,}4 \cdot 5 \quad ; \quad B_6 = \frac{(3 \cdot 3) + (2{,}4 \cdot 5)}{5} = \frac{3}{5} = 0{,}6\ t$ (OK)

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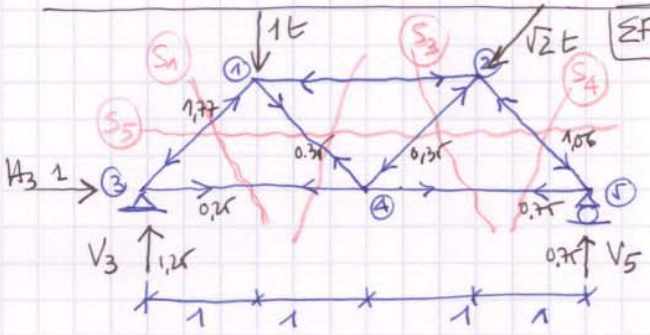



$\Sigma F_H = 0 \quad H_3 = 1\ t$

$\Sigma M_3 = 0 \quad (1 \cdot 1) + (1 \cdot 3) = (1 \cdot 1) + (V_5 \cdot 4)$

$V_5 = \frac{3}{4} = 0{,}75\ t.$

$\Sigma F_V = 0 \quad 1 + 1 = 0{,}75 + V_3 \Rightarrow V_3 = 1{,}25\ t$

SECCIÓ 1 $\quad \Sigma M_1 = 0 \quad (1 \cdot 1) - (1{,}25 \cdot 1) + (3.4 \cdot 1) = 0$

$-0{,}25 + 3.4 = 0 \Rightarrow 3.4 = 0{,}25\ t$

$H_3 = 1$
$V_3 = 1{,}25$
$V_5 = 0{,}75$

| | | |
|---|---|---|
| ✓ | 3.1 | 1,77 C |
| | 1.2 | 1,5 C |
| | 1.4 | 0,35 T |
| | 4.2 | 0,35 C |
| ✓ | 2.5 | 1,06 C |
| | 3.4 | 0,25 T |
| | 4.5 | 0,75 T |

$\Sigma M_4 = 0 \quad (3.1) \cdot \sqrt{2} = 1{,}25 \cdot 2 \Rightarrow (3.1) = \frac{2{,}5}{\sqrt{2}} = 1{,}77\ t$

$(3.1)\sqrt{2} + (1.4) + (0{,}75 \cdot 2) = 0 \Rightarrow 2{,}5 = (3.1)\sqrt{2} \Rightarrow (3.1) = \frac{2{,}5}{\sqrt{2}}$

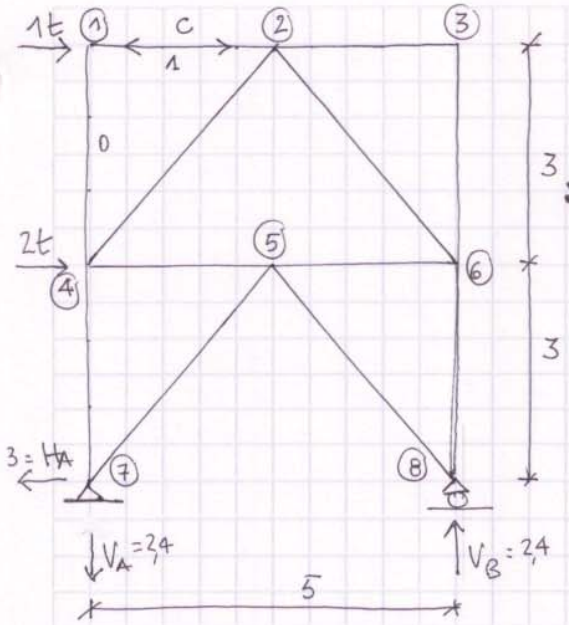
SECCIÓ 4 $\quad \Sigma M_4 = 0 \quad (0{,}75 \cdot 1) = (2.5) \cdot \sqrt{2} \Rightarrow (2.5) = \frac{0{,}75}{\sqrt{2}} = 1{,}06\ t$

$\Sigma M_2 = 0 \quad (0{,}75 \cdot 1) = (4.5) \cdot 1 \Rightarrow (4.5) = 0{,}75\ t$

SECCIÓ 5 $\quad \Sigma M_2 = 0 \quad (1 \cdot 2) - (1{,}77 \cdot \sqrt{2}) + (1.4) \cdot \sqrt{2} = 0 \quad ; \quad (1.4)\sqrt{2} = 0{,}5 \quad ; \quad (1.4) = 0{,}35\ t$

SECC.3 $\quad \Sigma M_1 = 0 \quad ; \quad -(2.4)\sqrt{2} + (0{,}75 \cdot 1) + (1 \cdot 1) - (1{,}25 \cdot 1) = 0 \quad ; \quad (2.4) = \frac{0{,}5}{\sqrt{2}} = 0{,}35\ t$

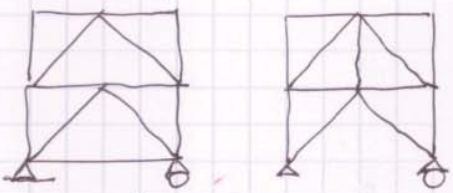
$\Sigma M_4 = 0 \quad ; \quad (1.2) \cdot 1 + (1.4) - (1{,}25 \cdot 2) = 0 \quad ; \quad (1.2) = 2{,}5 - 1 = 1{,}5\ t$



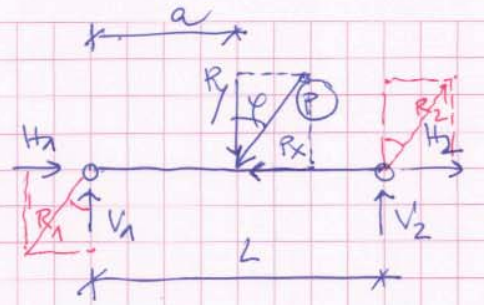

$V_B = \frac{(1\cdot 6)+(2\cdot 3)}{5} = 2{,}4t = V_A$ ; $H_A = 3t$

⇒ **ULL!**

mecanisme! ⇒



cal afegir ↕ o ↔ una barra




$\Sigma F_V = 0$
$\Sigma M_1 = 0$
← $V_1 ; V_2$

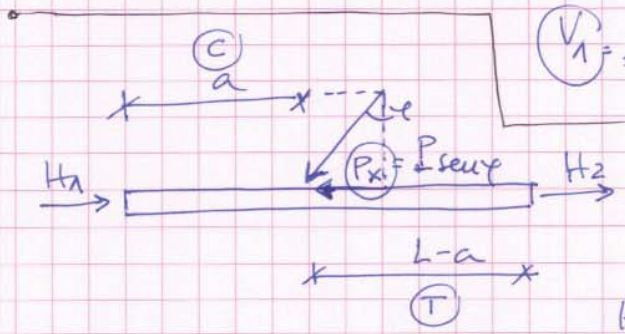
CARGAS FUERA DE LOS NUDOS

$V_1 + V_2 = P \cos\varphi$

$V_2 \cdot L - P_y a = 0$ ; $V_2 = \frac{P_y \cdot a}{L} = \frac{P \cos\varphi \cdot a}{L}$

$V_1 = P\cos\varphi - \frac{P\cos\varphi \cdot a}{L}$

$V_1 = P\cos\varphi \left(1 - \frac{a}{L}\right) = \frac{P\cos\varphi\,(L-a)}{L}$




$H_1 ; H_2$ → Equacions compatibilitat de deformacions
$\Sigma F_H = 0$

$H_1 + H_2 = P_x = P\sin\varphi$

$\Delta_C = \frac{H_1 \cdot a}{S E}$ ; $\Delta_t = \frac{H_2 (L-a)}{S E}$

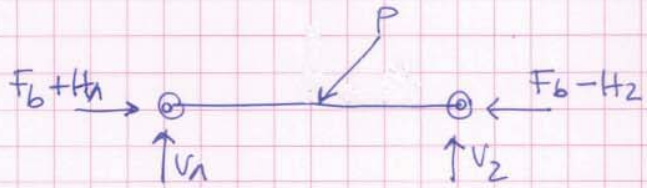
$\Delta_C = \Delta_t$ → $H_1 \cdot a = H_2 (L-a)$ ; $(P_x - H_2) a = H_2 (L-a)$

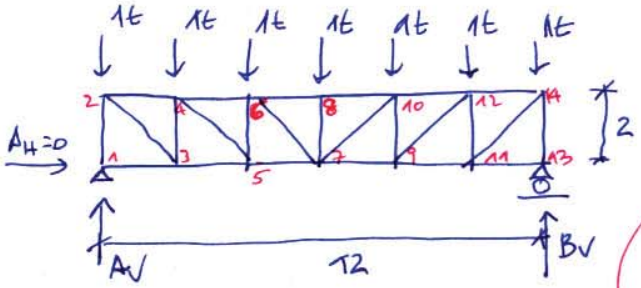
$P_x a - H_2 a = H_2 L - H_2 a$ ; $H_2 = \frac{P_x a}{L} = \frac{P\sin\varphi \cdot a}{L}$

$H_1 = P\sin\varphi - H_2 = P\sin\varphi \left(1 - \frac{a}{L}\right) = P\sin\varphi \left(\frac{L-a}{L}\right)$

- APLICAR EN LA ESTRUCTURA (NUDS) LES REACCIONS ① i ② CANVIADES DE SIGNE COM A CÀRREGUES FORA DELS NUDS.
- ES SUMARÀ L'ESFORÇ OBTINGUT AMB EL DE LA PRÒPIA ESTRUCTURA QUE VE DELS NUDS. ($F_b$)
- TAMBÉ, SI HEM DE DIMENSIONAR LES BARRES → VOLEM SABER AMB CÀRREGUES INTERMITGES ESTAN SOTMESES A FLECTOR I TALLANT.

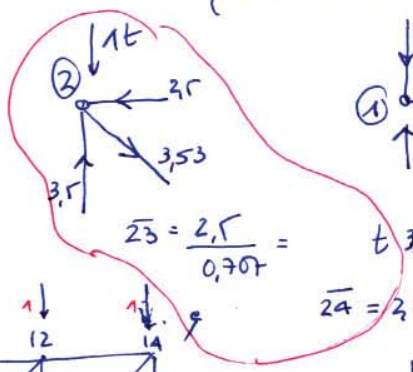
EX: SI ES UNA BARRA COMPRIMIDA.






$A_V + B_V = 7\,t \Rightarrow$ simetria $A_V = B_V = 3{,}5\,t$.

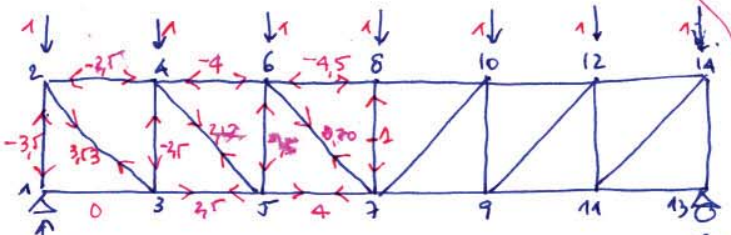
$\beta = 45° \rightarrow \cos 45° = \operatorname{sen} 45° = 0{,}707$

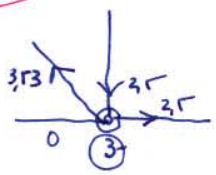



$\bar{12} = 3{,}5\,t$ (C)(−)

$\bar{23} = \frac{2{,}5}{0{,}707} = 3{,}53\,t$

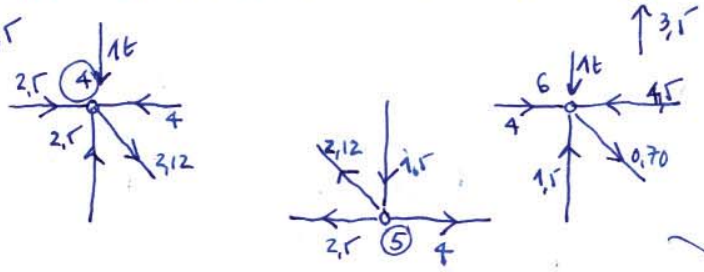
$\bar{24} = 2{,}5\,t$




$\bar{34} = 1{,}5\,t$ (C)(−)

$\bar{35} = 1{,}5\,t$ (T)(+)




**CORDÓ SUPERIOR**

$\bar{24} = \bar{1214} = -2{,}5\,t$

$\bar{46} = \bar{1012} = -4\,t$

$\bar{68} = \bar{810} = -4{,}5\,t$

**CORDÓ INFERIOR**

$\bar{13} = \bar{1113} = 0\,t$

$\bar{35} = \bar{911} = 2{,}5\,t$

$\bar{57} = \bar{79} = 4\,t$

**MUNTANTS**

$\bar{12} = \bar{1314} = -3{,}5\,t$

$\bar{34} = \bar{1112} = -2{,}5\,t$

$\bar{56} = \bar{9\text{-}10} = -1{,}5\,t$

$\bar{78} = -1\,t$

**DIAGONALS**

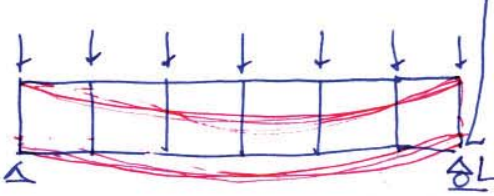
$\bar{23} = \bar{1114} = 3{,}53\,t$

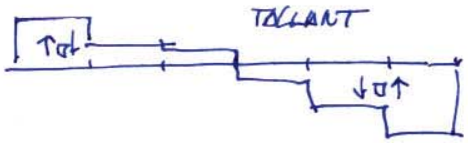
$\bar{45} = \bar{912} = 2{,}12\,t$

$\bar{67} = \bar{710} = 0{,}70\,t$

**CONCLUSIÓ:**

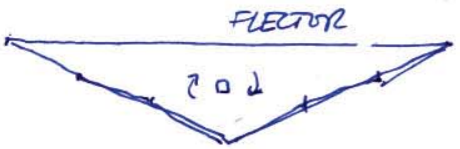
- MUNTANTS: - treballen a compressió
  - augmenta la tensió al extrem / major tallant
- CORDÓ SUPERIOR: treballa a compressió
  - augmenta la tensió al centre / major flector (+)
- CORDÓ INFERIOR: treballa a tracció
  - augmenta la tensió al centre / major flector (+)
- DIAGONALS: treballen a tracció
  - augmenta la tensió als extrems / tallant major (+)






muntants → comprimit en Q màx/maj

diagonals → traccionats en Q màx/maj




cordó superior [ comprimit en Mf (+) / traccionat en Mf (−) ]

cordó inferior [ comprimit en Mf (−) / traccionat en Mf (+) ]